

4.4

SOLVE  $y'' + y = \sin x \longrightarrow y'' + y = 0$

$$y_p = (A \sin x + B \cos x)x$$

$$r^2 + 1 = 0$$

$$r = \pm i$$

$$y_h = C_1 \sin x + C_2 \cos x$$

$$y_p = Ax \sin x + Bx \cos x$$

$$y_p' = -Bx \sin x + A \sin x + Ax \cos x + B \cos x$$

$$= (-Bx + A) \sin x + (Ax + B) \cos x$$

SANITY CHECK:

COEF OF  $\sin x$  LOOKS LIKE  
COEF OF  $\cos x$  WITH A, B

SWAPPED + SOME SIGNS  
ALTERED

$$y_p'' =$$

$$(A-B) \sin x + (A-B)x \cos x + A \cos x$$

$$(-A-B)x \sin x - B \sin x$$

$$(A+B) \cos x$$

$$y_p'' = ((-A-B)x + (A-2B)) \sin x + ((A-B)x + (2A+B)) \cos x$$

$$+ y_p$$

$$Ax \sin x$$

$$+ Bx \cos x$$

$$= (-Bx + (A-2B)) \sin x + (Ax + (2A+B)) \cos x$$

ERROR:

$x \sin x, x \cos x$

IN  $y_p$  AFTER ADJUSTING  
FOR OVERLAP WITH  $y_h$   
SHOULD HAVE VANISHED

$$y_p'' = -B \sin x - Bx \cos x + A \cos x$$

$$-Ax \sin x - B \sin x + A \cos x$$

$$y_p'' = (-\cancel{Ax} - 2B) \sin x + (-\cancel{Bx} + 2A) \cos x$$

$$+ y_p' + (\cancel{Ax}) \sin x + (\cancel{Bx}) \cos x$$

$$= -2B \sin x + 2A \cos x = \sin x$$

$$-2B = 1 \quad \text{AND} \quad 2A = 0$$

$$B = -\frac{1}{2} \quad A = 0$$

$$y_p = -\frac{1}{2} x \cos x$$

$$y = -\frac{1}{2} x \cos x + C_1 \sin x + C_2 \cos x$$

$$y'' - 2y' + y = 8e^x + xe^x$$

$$r^2 - 2r + 1 = 0 \rightarrow r = 1, 1$$

$$y_h = c_1 e^x + c_2 x e^x$$

$$y'' - 2y' + y = 8e^x \quad \text{POLY DEGREE 1} \quad \text{EXP } e^{kx}$$

$$y'' - 2y' + y = xe^x$$

$$y_{p1} = Ae^x$$

$$y_{p2} = Axe^x + Be^x$$

$$(Ax+B)e^x$$

POLY DEGREE 1

$$6A = 1 \quad 2B = 8$$

$$A = \frac{1}{6} \quad B = 4$$

$$y_p = \frac{1}{6}x^3e^x + 4x^2e^x$$

$$y = \left(\frac{1}{6}x^3 + 4x^2 + c_2x + c_1\right)e^x$$

$$y_p = (Ax^3e^x + Bx^2e^x)$$

$$= Ax^3e^x + Bx^2e^x \quad \leftarrow \text{FORM OF } y_p$$

$$y_p' = Ax^3e^x + 3Ax^2e^x$$

$$+ Bx^2e^x + 2Bxe^x$$

$$= Ax^3e^x + (3A+B)x^2e^x + 2Bxe^x$$

$$y_p'' = Ax^3e^x + 3Ax^2e^x$$

$$(3A+B)x^2e^x + (6A+2B)xe^x$$

$$+ 2Bxe^x + 2Be^x$$

$$y_p'' = Ax^3e^x + (6A+B)x^2e^x + (6A+4B)xe^x + 2Be^x$$

$$-2y_p' \quad -2Ax^3e^x + (6A-2B)x^2e^x \quad -4Bxe^x$$

$$+y_p \quad Ax^3e^x \quad + Bx^2e^x$$

$$6Axe^x + 2Be^x = 8e^x + xe^x$$

FIND THE FORM OF  $y_p$  FOR  $y'' + 2y' = x^2(1 - e^{-2x})$

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DON'T SOLVE FOR  
THE ACTUAL COEF'S